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THE RELATION BETWEEN PLUMB-LINE DEFLECTION, GRADIENTS, AND RADII OF CURVATURE OF A GEOID

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Summary: A method is given for deducing the formulas for studying the geoidal surface with the aid of gradients of the terrestrial gravitational field.

I

We shall write two equations serving to determine the component plumb-line deflections or deviations in the main planes: x and y, coinciding respectively with the meridian and the primary vertical (normal):

$$\xi = -\frac{T_x}{g}, \qquad \eta = -\frac{T_y}{g} \qquad (1)$$

where $T_x = \frac{dT}{dx}$ and $T_y = \frac{dT}{dy}$ are the first derivatives of the disturbing potential T and g is the force of gravity at the surface of the geoid.

With the aid of these two equations, we shall find the analytical relation between: the component plumb-line deflections ξ and η, the second derivatives $T_\Delta = \frac{d^2T}{dx^2} - \frac{d^2T}{dy^2}$ and $T_{xy} = \frac{d^2T}{dxdy}$, and the radius of curvature R of the level surface of a geoid:

$$W(x, y, z) = C$$

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Differentiating (1), respectively, with respect to x and y, we obtain

$$d\xi = -\frac{1}{g}\left[T_{xx} + T_{xy}\,\mathrm{tg}\,A\right] dx,$$
$$d\eta = -\frac{1}{g}\left[T_{yy} + T_{xy}\,\mathrm{ctg}\,A\right] dy, \tag{2}$$

where A is the azimuth of the element of length $dl = \sqrt{dx^2 + dy^2} = \frac{dx}{\cos A} = \frac{dy}{\sin A}$

If the first equation of (2) is multiplied by sin A and the second equation by cos A, then after subtracting one equation from the other we obtain:

$$d\xi \sin A - d\eta \cos A = T_{xy}^{(l)} dl, \tag{3}$$

where

$$T_{xy}^{(l)} = \frac{1}{2} T_\Delta \sin 2A + T_{xy} \cos 2A, \tag{4}$$

hence, after integration, we obtain Eötvös' equation:

$$(\xi_2 - \xi_1) \sin A - (\eta_2 - \eta_1) \cos A = \frac{1}{g}\int_1^2 T_{xy}^{(l)} dl, \tag{5}$$

which finds application in the study of geoidal surfaces.

III

After the equations of (2) are multiplied respectively by cos A and sin A, we add them term by term. As a result of a not inconsiderable number of transformations, we obtain:

$$d\xi \cos A + d\eta \sin A =$$
$$= -\frac{1}{g}\left[T_{xx}\cos^2 A + T_{yy}\sin^2 A + 2T_{xy}\sin A\cos A\right] dl, \tag{6}$$

hence, after integrating in the same interval as used previously, we obtain:

$$(\xi_2 - \xi_1)\cos A + (\eta_2 - \eta_1)\sin A =$$
$$= -\frac{1}{g}\int_1^2 \left[T_{xx}\cos^2 A + T_{yy}\sin^2 A + 2T_{xy}\sin A\cos A\right] dl. \tag{7}$$

The equation just obtained above may be employed to determine the radius of curvature of a level surface of a geoid. Thus, it is sufficient to turn our attention to the nature of the integrand in function (7). It represents the negative quantity of the product of surface curvature $\left(\frac{1}{R}\right)$ multiplied by the acceleration of the force of gravity (g), that is to say it equals (2):

$$-\frac{g}{R} = T_{xx}\cos^2 A + T_{yy}\sin^2 A + 2T_{xy}\sin A\cos A. \tag{8}$$

Consequently, instead of (6), we have:

$$\frac{d\xi}{dl}\cos A + \frac{d\eta}{dl}\sin A = \frac{1}{R}, \tag{9}$$

which gives, in the planes of the meridian and of the primary vertical (normal), the equalities:

$$\frac{d\xi}{dl} = \frac{1}{R_M}, \quad \frac{d\eta}{dl} = \frac{1}{R_N}. \tag{10}$$

We note, for clarity, that $R_L \neq$ and $R_N \neq N$ [2]. The expression (7) determines the deviation of the curvature of the geoid from the curvature of the surface of reference (deviation). Therefore, in order to obtain the complete curvature of a geoid, it is obviously necessary to calculate also the curvature of the surface, relative to which the geoidal surface is studied.

IV

We return again to the equations in (2) and we set $A = \alpha + (A - \alpha)$, where α is the azimuth of the vector of plumb-line deflection $\varepsilon = \sqrt{\xi^2 + \eta^2}$. Hence, for the case $A - \alpha = 0$, we obtain:

$$d\xi = -\frac{1}{g}\left[T_{xx} + T_{xy}\,\mathrm{tg}\,\alpha\right]dx, \qquad (11)$$

$$d\eta = -\frac{1}{g}\left[T_{yy} + T_{xy}\,\mathrm{ctg}\,\alpha\right]dy,$$

Hence, keeping in mind that:

$$d\xi = d\varepsilon \cdot \cos\alpha, \quad \text{and} \quad dx = dl \cdot \cos\alpha,$$

$$d\eta = d\varepsilon \cdot \sin\alpha, \qquad\qquad dy = dl \cdot \sin\alpha,$$

we obtain:

$$T_{xx} + T_{xy}\,\mathrm{tg}\,\alpha = T_{yy} + T_{xy}\,\mathrm{ctg}\,\alpha. \qquad (12)$$

Consequently,

$$\mathrm{tg}\,\alpha - \mathrm{ctg}\,\alpha = \frac{T_\Delta}{T_{xy}}, \quad \frac{d\xi}{d\eta} = \mathrm{ctg}\,\alpha. \qquad (13)$$

Setting:

$$\mathrm{tg}\,\alpha - \mathrm{ctg}\,\alpha = -\frac{2}{\mathrm{tg}\,2\alpha}, \qquad (14)$$

we obtain:

$$\mathrm{tg}\,2\alpha = -\frac{2T_{xy}}{T_\Delta}. \qquad (15)$$

Equation (15) determines α in the direction of the plumb-line deflection ε if T_Δ and T_{xy} are known.

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